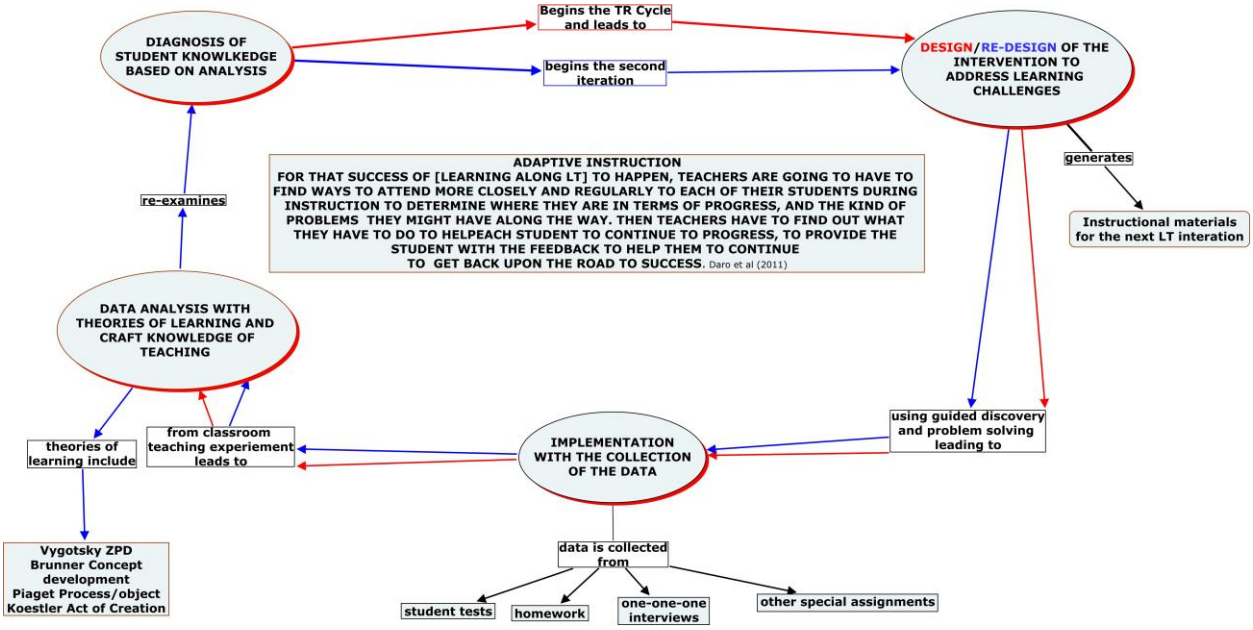
Teaching-Research NYCity model



Teaching – Research Cycle

Teaching-Research NYCity Model

Czarnocha, Prabhu,2006, Dydaktyka Matematyki

It is the classroom investigations of learning processes conducted simultaneously with teaching by the classroom teacher, aimed at the improvement of learning in the classroom, and beyond.

On the shoulders of the giants:

Lewin, 1946

Action Research and Minority Problems, Journal of Social Issues 2

"It is important to understand clearly that social research concerns itself with two rather different types of questions, namely the study of general laws of group life and the diagnosis of a specific situation."

Vygotsky

"to catch the processes in their development and to determine how instruction can optimally influence these processes" Kantowski,1979

Koestler, 1964,

The Act of Creation

I have coined the term "bisociation" in order to make a distinction between the routine skills of thinking on a "single plane" as it were and the creative act, which as I shall try to show, always operates on more than one plane" [at least two planes].

National Academy, USA

How People Learn, 1999

Research directly influences classroom practice when teachers and researchers collaborate in design [teaching] experiments or when interested teachers incorporate ideas from research into their classroom practice

Theories of Learning and TR Cycle

Action, Process, Object, Schema APOS, Dubinsky
(Asiala et al, 1997, Research in Collegiate Mathematics Education 2,MAA)

: *A step-by-step action becomes conceptualized as a total process [and] is encapsulated as a mental object…The final part of the APOS structure occurs when "actions processes and objects…are organized into structures which we refer to as schemas"* (Tall et al,2000). The process through which a schema becomes a cognitive object is called thematization (Piaget, 1995) that is involving the schema in different problem situations, so that all its components and transformations between them are clearly perceived and assimilated.

Reification Theory, A. Sfard.

: "*first there must be a process performed on the already familiar objects, then the idea of turning this process into a more compact, self-contained whole will emerge, and finally an ability to view this new entity as a permanent object must be acquired. These three components of concept development will be called interiorization, condensation and reification* " (Sfard 1992,pp.64-65).

The Triad of Piaget and Garcia (1989)
is a mechanism of thinking leading to concept formation formulated on the basis of the thorough comparative analysis of the development of physical and mathematical ideas in history of science on one hand, and the psychogenetic development of these concepts in a child, on the other (PG,1989). It is defined as the passage through intra-operational, inter-operational and trans-operational stages. "*Intra-operational stages are characterized by intra-operational relations, which manifest themselves in forms that can be isolated*"
inter-operational stage is "*characterized by correspondences and transformations among the forms that can be isolated at previous levels…*"
"*The trans-operational stages are characterized by the evolution of structures whose internal relationships correspond to inter - operational transformations*."

Design: THREE STRATEGIES TO SOLVE RATES PROBLEMS (Iteration 1)[1]

Three concepts: R – unit rate; T-total amount, N – number of units.

Strategy #1. (Given R,N. Unknown T)

Juan is making $24/hour as a carpenter**.**

1. ***How much in total will he make in 1, 2, 7, N hours?***

Total T $	Calculations with the unit rate R = $24/hr	Number of hours
		1
		2
		7
		N

Thinking reflection questions:

Recall the steps of the calculations you made above, thoughtfully look into the numbers in the table and answer following questions.

2. ***If you know the number N of hours Juan works, how would you calculate his total pay?.....***
3. Now look back into last two problems, compare the steps of calculations and answer the question: ***If the total pay is T, $/hr is the rate R, and hours of work are N, how would you write the correct general formula governing this problem?.....***

APPLICATION EXERCISES

To the student: Read every problem carefully and decide which strategy you will use to solve it.

1. **Exercise** It costs Lovell $2400 for 12 credits at a community college. Find the cost per credit that he is paying.

2. Exercise If Jorge jogs 25 km in 2.5 hours, what speed was he jogging?

3. Exercise The deer runs towards the path in the forest, which is at a distance of 1000 yards. If the animal runs with the speed of 125 yards per second, how many seconds will it take for the deer to get to the forest?

4. Exercise Travis goes roller blade skating at a rate of 8 ¼ miles per hour for $\frac{2}{3}$ of an hour. How far does he go?

5. Exercise Sheila's pick up truck get 16 miles per gallon of gasoline, at this rate how far can she drive with $5\frac{3}{4}$ gallons of gasoline?

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[1] The full design contains three such teaching sub-sequences, each for one of three three representation of the formula

$T_{otal} = R_{ate} \times N_{umber\ of\ units}$

6. Exercise. Juana typed 225 words in 7 ½ minutes. How many words did she type per minute?
7. Exercise: If you knew the rate in words per minute that Tanya was typing her Humanities term paper and you knew how many minutes she typed, how would you find the number of words she typed?
8. Exercise Juan typed a paper of 640 words with the speed of 40 words/minute. How many minutes did that typing take?

ANALYSIS OF THE DATA

- Two students, instead of replying with majority stating " $24 x N" as the answer, used the additive structure stating: "you have to add them all up". It seems that those two students are at as-yet unfinished process of "condensation" to the second level of understanding. The absence of student understanding of multiplication as repeated addition creates a cognitive obstacle for the creation of the "compact self-contained whole" here needed, accordingly to (Sfard, 1992), to progress along the developmental trajectory.
- Two other students displayed still more elementary mathematical behavior responding to the same questions; instead of reasoning with the variable N to answer the question, they assumed a concrete value (N = 13) and calculated the total based on this value. For them, the concept of the rate didn't progress beyond the first stage of development of using it only in concrete, elementary cases.
- Use of learning theories in the design of activities was fruitful; it has shown that (a) there were students present at each level of the three levels of concept development, and (b) the adaptive instruction strategies to help students to reach the full understanding of the rate concept could be determined on their basis. The two students in the first level need strong reinforcement in the variable as a generalization of a number, while the students placed in transition between levels need the reinforcement of multiplication as repeated addition.

- An additional relevant information was provided by students' results in the departmentally designed final exam, whose results could be divided into standard three subgroups: top 20%, middle 40%, and bottom 40% failing it. The two students placed by the assignment in the first level of development failed the exam, while the two students who were placed along the transition between the first and second level passed the exam, placing themselves in the second subgroup.

Second Iteration

Design: Rates Teaching Sequence – 2nd iteration.

THREE STRATEGIES TO SOLVE RATES PROBLEMS

Three concepts: R – unit rate; T-total amount, N – number of units.

THREE STRATEGIES TO SOLVE RATES PROBLEMS (Iteration 2)

Three concepts: R – unit rate; T-total amount, N – number of units.

Strategy #1. (Given R,N. Unknown T)

Juan is making $24/hour as a carpenter**.**

1.1 ***How much in total will he make in 1, 2, 7, N hours?***

Total T $	Calculations with the unit rate R = $24/hr	Number of hours
		1
		2
		7
		N

Thinking reflection questions:

Recall the steps of the calculations you made above, thoughtfully look into the numbers in the table and answer following questions.

1.2 If you know the number N of hours Juan works, how would you calculate his total pay?.....

1.3 Juan got a raise to $30/hr. ***If you know the number N of hours Juan works, how would you calculate his total pay now?.....***

……..

1.4 Now look back into last two problems, compare the steps of calculations and answer the question: ***If the total pay is T, $/hr is the rate R, and hours of work are N, how would you write the correct general formula governing this problem?.....***

1.5 How would you convince a teacher you are right?.....

Strategy #2 (R, T-known. Unknown N)

2.1 Michelle's car gets 40 mi/gal, and she has to drive 80 miles total to see a friend.

How many gallons of gas does she have to buy? How many gallons if she has to drive

2.2 She will meet there another friend who had traveled 160 miles in a similar car burning 40mi/gal. How many gallons did that friend use?

2.3 If your daughter has a trip of A miles to make, describe in words how will she calculate the number of gallons of gas needed if her car makes 50mi/gal?.....

2.4 Describe in words how will you calculate the number of gallons if she has 200 miles to drive and her car makes X miles/gallon?.....

Total miles T	Unit rate R=miles/gallon Show the computation in each row	Number of gallons N
80 miles	40miles/gal	

160miles	40miles/gal	
A	50miles/gal	
200 miles	X miles/gallon	

2.5 a) Now look back onto last two problems. Compare the steps of calculations and answer the question: ***If the total pay is T, $/hr is the rate R, and hours of work are N, how would you write the correct general formula governing these problems?.....***

b) Compare your method of calculation while using Strategy 1 to your method using Strategy 2. What is similar and what is different?……
……..
c) In what situations will you use Strategy 1 and when Strategy 2……..
…….
Annotations: *The series of questions 2.5 a,b,c starts to develop the relationship between different strategies.*

Strategy #3 Calculation of the rates. (T,N-known. Unknown: R)

This strategy applies whenever we want to find the unknown rate and we know both the Total and the Number of units N. Solve those problems and write the answers in the table below.
Exercise 3.1 You are earning $1000 in 4 weeks. What's your unit rate in $/week? (How much do you make per week?)
Exercise 3.2 A worker is making $2400 in 3 months. How much is he getting per month?
Exercise 3.3 Juanita is making $D in 5 hours. Write how would you calculate her $/hour?……..
……
Exercise 3.4 Marco, the salesman, sold S shirts for $200. How would you calculate the cost of 1 shirt?
……

Total, T	R-Unit Rate. Show your calculations and write in their answers with the correct units of the unit rate	N Number of units
$1000	….$/week	4 weeks
$ 2400	….$/…….	3 months
$ D	…..$/…….	5 hours
$ 200	….$/shirt	S shirts

3.5) Now look back to the last two problems. Compare the steps of calculations and answer the question: ***If the total pay is T, $/hr is the rate R, and hours of work are N, how would you write the correct general formula governing these problems?.....***

3.6 (Solve in the table below**). Annotations**: *Table 3.6 generalizes the strategy to obtain different categories of rates by the same technique.*
A runner runs 8 km in 2 minutes. What's his unit rate in km/min?
A car trip of 250 miles used 5 gallons of gas. How many miles did this car use for the trip.?
Registering for classes you paid $1200 for C credits. Describe in words how would you calculate the cost of one credit ?

Total	Unit rate R. Show your calculations and the units of R	Number of Units
8 km		2 min
250 miles		5 gallons

$1200		C credits
540 words of typing		M minutes

Annotations: *Questions 3.7-3.9 aim at developing student problem solving schema through the strategy choice method*.

3.7 Compare your method of calculation while using the Strategy 1, Strategy 2 and the Strategy 3. What is similar and what is different in all of them?……
……..

3.8 When will you apply strategy 1, strategy 2 and strategy 3…….

3.9 Fill up the missing cells in the table below; solve the following problems by applying correct strategy 1,2 or 3.

Unit rate R	Total T	Number of units N	# of the strategy
7 ft/min		12 minutes	
	650 typed words	25 minutes	
12.5 miles/hr	250 miles		
	5000 miles	250 gallons	
	192 oz	12 lb	
3 feet/yard		11 yards	

Annotation: *The series A,B,C develops student use of decimals and fraction in each of the contexts – standard challenge for students in remedial mathematics.*

Series A

A1. A car travels 5 hours at the speed of 80 km/hr. How far will it go?
A2 A car travels 3.8 hours at the speed of 60.7 km/hr. How far will it go?
A3 A car travels 4 ¾ hours at the speed of 52 km/hr. How far will it go during this time?

Series B

B.1.A car travelled 540 miles in 9 hours. What was the speed of the car? How far could he go in 15 hours?

B 2. A car travelled 104 miles in $3\frac{1}{4}$ *hrs*. What was its speed? How far did it go in $4\frac{3}{8}$ *hrs*?

B.3. A car travelled 530.6 mi in 7 hours. What was his speed? How far did it travel in 8.5 hours?

Series C

C.1. A nurse is makes $25/hr. She makes $300 during her night shift. How many hours is her night shift?
C.2. An apprentice bricklayer earns $18 & 1/3 per hour. How many hours does he have to work to make $75?
C.3. A tutor tutors for C hours weekly, making $30.6/hr. How many hours does she have to work to make $157.72.

Annotation: *The aim of the following section is to develop mastery in transformation of the rate basic formula followed by its application in complex situations. It is expected that students will encapsulate their understanding of rates as the schema of the single formula.*

Syntheses of 3 Strategies into one formula: $T_{otal} = R_{ate} \times N_{umber\ of\ units}$,

Write your previous answers/formulas to the questions:
Strategy 1 1.4……..
Strategy 2 2.5a……..
Strategy 3 3.5……..

Fill out the table:

Transformation of strategies	Word description. What will you do to transform the formula?	Computation	Target, final formula/strategy
1→2?			
1→3?			
2→1?			
2→3?			
3→1?			
3→2?			

Application of the rate formulas into complex problems solving.

- I make $50/hr during the first 8 hours of work. I received an hourly increase for the next 5 hours of work. All together I received $725 for 14 hours of work. What was my hourly pay increase for the last 5 hours of work? What was my percent increase?
- A typist typed 40w/min during the first session of typing. During the second session of typing, which lasted 30 min, the typist typed 60 w/min. All together she typed 2600 words. How long was the first session?
- David and his wife are driving from work to his parent's house. That trip takes them 2 hours and 30 minutes if he drives 60 mph. David knows there is a good diner at exit 24 which is about 75% of the distance from work to his parent's house. About how far is it to the diner from their jobs?

ITERATION AS THE METHOD FOR CONSTRUCTION OF LEARNING TRAJECTORIES THROUGH CLASROOM TEACHING – RESEARCH

(submission to PME 37,Germany)

Engagement of LT framework in the development of curriculum is facilitating intense discussions on the fruitfulness of the relationship between of research and teachers in the support of the Common Core effort. "*Whose responsibility is it to construct learning trajectories?*" asks (Steffe, 2004, p.130). (Battista, 2004p.188) informs "to implement instruction that genuinely and effectively supports student construction of mathematical meaning and competence *teachers must not only understand cognition-based research on students' learning, they must also be able to use that knowledge to determine and monitor the development of their own students' reasoning*."(Empson,S. 2011) adds a layer of complexity to our research on learning and invites us to think seriously about how to support teachers to incorporate knowledge of children's learning into their purposeful decision-making about instruction. (Clements and Sarama, 2004,p.85) note "*that learning trajectories could and should be re-conceptualized or created by small groups or individual teachers, so that there are based on more intimate knowledge of the particular students involved…*"

Thus, in agreement with (Kieran et al, 2013) "*it is* [only] *the teacher who can affect to the greatest extent the achievement of one of the main purposes of the research enterprise, that is, the improvement of students' learning of mathematics*"; therefore, the search is on for the most effective routes of joining educational research with classroom teaching.

Learning Trajectory. Definitions.

The number of different definitions of this concept abundant in the literature suggests that its ontological and epistemological nature is far from clear.

*A **Hypothetical** Learning Trajectory consists of the [mathematical] goal of student learning, the Mathematical Tasks that will be used to promote student learning, and hypotheses about student learning* (Simon,1995).

*[**Actual**] learning trajectory of children includes a model of their initial concepts and operations, an account of observable changes in those concepts and operations...and the account of mathematical interactions that were involved in these changes* (Steffe, L, 2004).

For Clements and Sarama (2009) learning trajectories have three parts: *a goal (that is an aspect of a mathematical domain children should learn), a developmental progression, or a learning path through which children move through levels of thinking, and the instruction that helps them to move along that path.(p.17)*

However, in the paper by the same authors (Clements and Sarama, 2004) a more complex definition appears: "*the simultaneous consideration of mathematics goals, models of children's thinking, **teachers' and researchers'** models of children's thinking, sequences of instructional tasks, and the interaction of these at a detailed level of analysis of processes*" (p. 87)

Confrey and colleagues (2009) defined them as *"**researcher** conjectured, empirically-supported description[s] of the ordered network of experiences a student encounters through instruction … in order to move from informal ideas … towards increasingly complex concepts over time"* (p. 2). Articulating.

Thus, in agreement with (Kieran et al, 2013) "*it is* [only] *the teacher who can affect to the greatest extent the achievement of one of the main purposes of the research enterprise, that is, the improvement of students' learning of mathematics*"; therefore, the search is on for the most effective routes of joining educational research with classroom teaching.

Teaching-Research approach

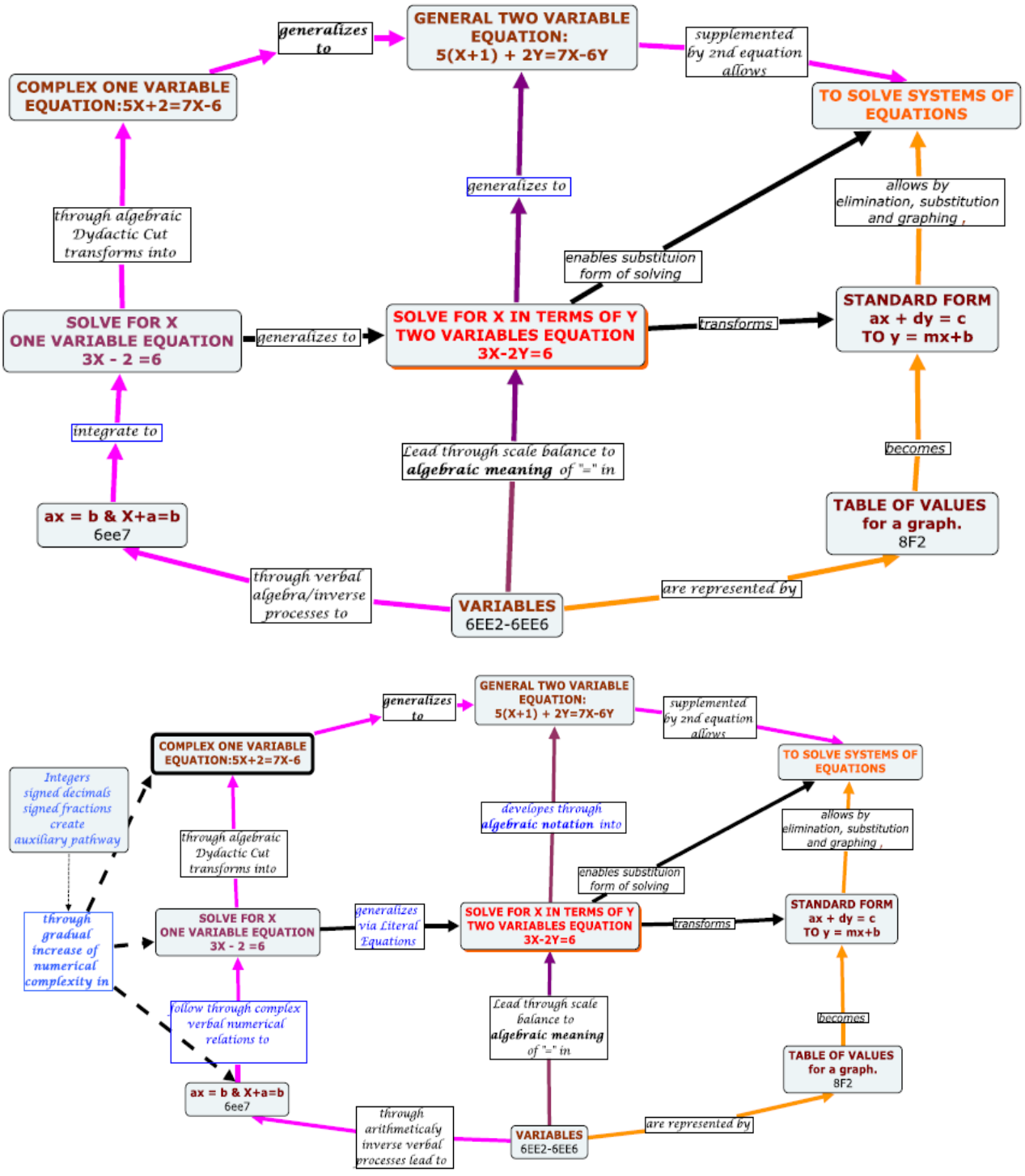
The design of **Learning Trajectory for Linear Equations** is the adaptive response to the observed challenges of students with the problem: *Solve for y in terms of x: 3x – 2y = 6.*

Student recorded solution: $3x - 2y = 6$

$\underline{-3x}$

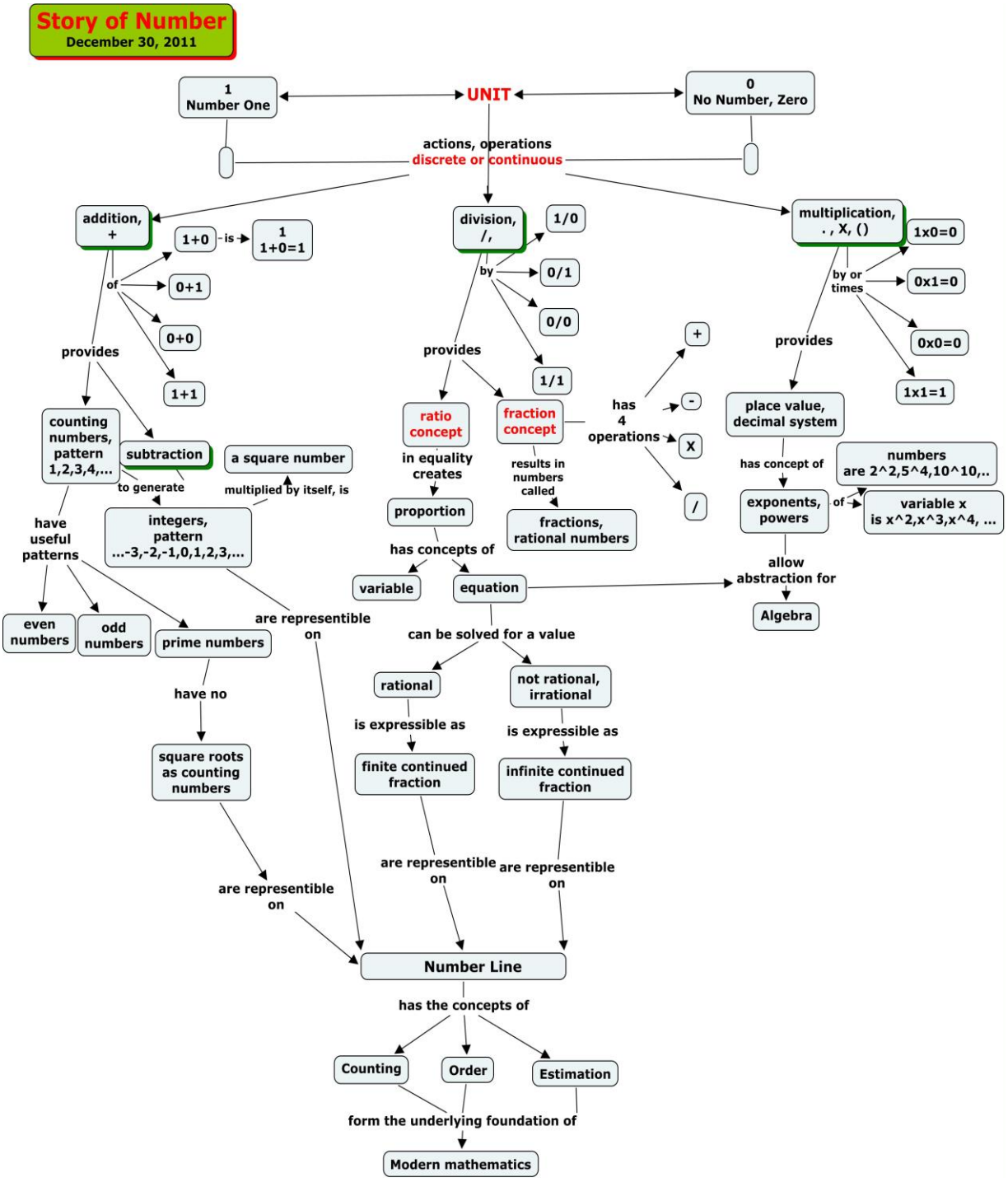
$-2y = 6$

$y = -3$



Two iteration of the Linear Equation Learning Trajectory. Two Iterations.

Design of learning trajectory through the full syllabus of the course (V.Prabhu)



Teaching – Experiment: The Flow of Thought Across the Zone of Proximal Development between Elementary Algebra and Intermediate English as a Second Language.

(Czarnocha, Prabhu, Pujol, PME 24, Japan)

A point of view that gives justice to the richness of relationships between thought and language might perhaps be found in (early) Vygotsky ([13]). There, thought and language are seen as being in a "reciprocal relationship of development." [8]. "Communication presupposes generalization…, and generalization… becomes possible in the course of communication" (p.7, [13]) – in other words, in order to communicate we need to think; and in order to think, we need to communicate. Such a view opens, in a very natural way, the possibility that thought - in our case, mathematical thought - might be able to shape natural language. One of the ways through which this process can take place is across the Zone of Proximal Development (ZPD). (Ch.6, [13]).

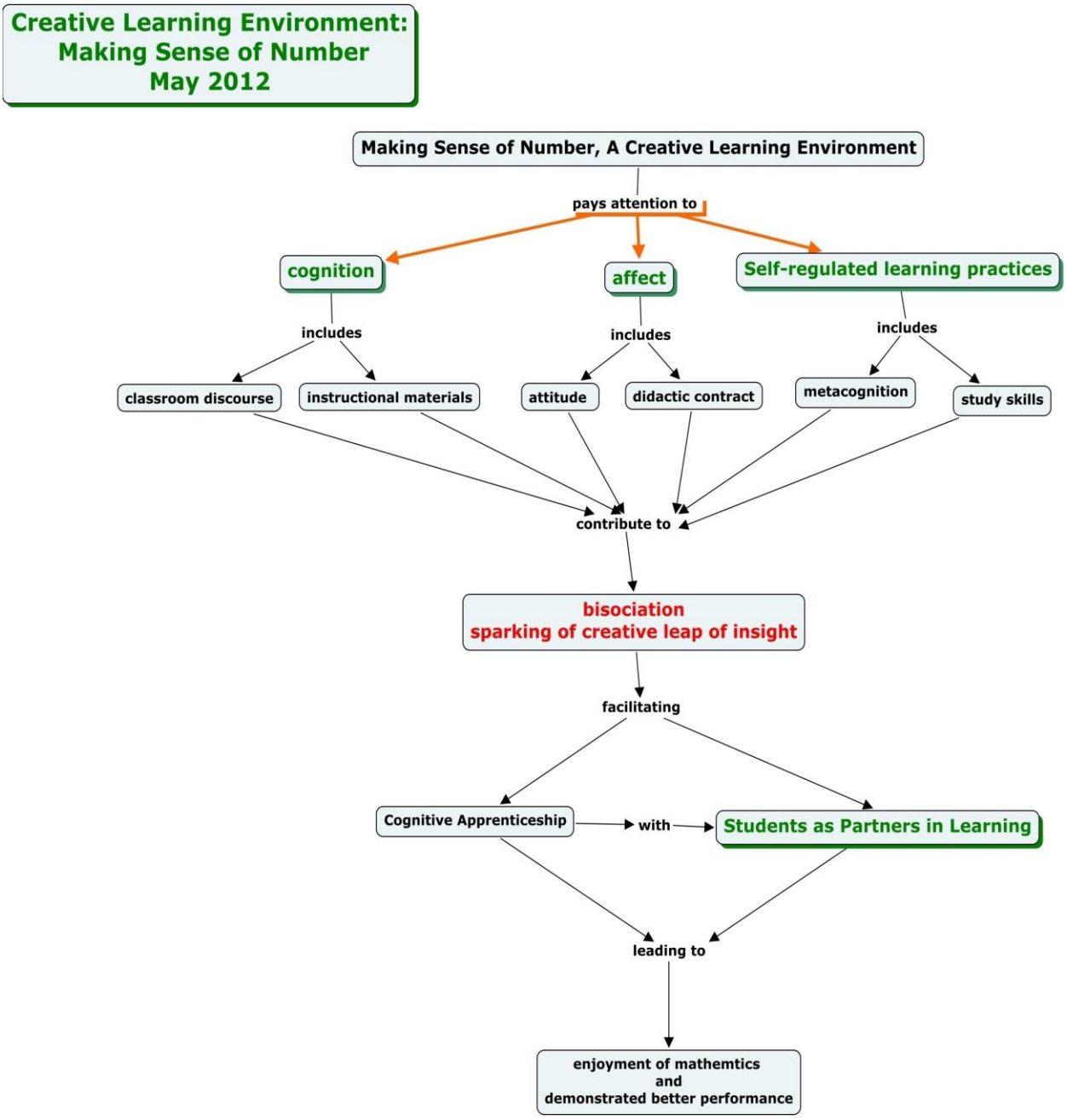
To confirm Vygotsky's highly dialectical view one would clearly need to detect the presence of two directions of developmental progression: the acquisition of English under the influence of mathematical thinking and the acquisition of mathematical understanding under the influence of the use of English. While the first direction is the main topic of this presentation, let us mention that the existence of the second has been confirmed, for the first time, in a recent experiment by Wahlberg [13]. Measuring the increase in the students' understanding induced by essay-writing; she observed a substantial (80%) increase in the experimental group as compared to the control group.

"Written speech assumes much slower, repeated mediating analysis and synthesis, which makes it not only possible to develop the required thought, but even to revert to its earlier stages, thus transforming the sequential chain of connections in a simultaneous, self-reviewing structure. Written speech thus represents a new and powerful instrument of thought"[2]

The judgment of the ESL instructor after reading all the essays of the experimental group was that they were more cohesive. As cohesiveness is closely related to the use of connectors - words such as "because", "yet", "although", etc.- all the connectors used by all students in their essay were categorized, counted, averaged per 22-line page, and the results compared with the corresponding numbers from the control group. _Our conclusion was that there was an average 15% increase in the number of connectors and subordinating clauses in the essays

[2] Luria, A.R., Yudovich, F. (1971) Speech and the development of mental processes in the child. Baltimore, MD: Pinguin

of the experimental group. This confirmed the ESL instructor's assessment that the long-term essays of the experimental group were more cohesive than the essays of the students who did not participate in the instructional link under discussion.



Teaching-Research as Transformative Research

Transformative research involves ideas, discoveries, or tools that radically change our understanding of an important existing scientific or engineering concept or educational practice or leads to the creation of a new paradigm or field of science, engineering, or education. Such research challenges current understanding or provides pathways to new frontier.

Teaching-Research NYCity Model (TR/NYC) is the investigation of classroom learning conducted simultaneously with teaching with the explicit aim to improve learning in the same classroom, and beyond. It helps to transform the classroom of every teacher of mathematics into "scientific educational laboratory" for the teacher, and therefore for the students. The practice of teaching-research creates a new mental attitude in which a novel design of instruction is at the same time an investigative probe into student thinking on the basis of which further teaching and research decisions are made. The increased degree of flexibility created by this collaboration between teaching and research within an individual helps teachers to reach new levels of instructional adaptability to student learning needs.

The revolutionizing aspect of this transformation is simple: the teaching-research, as the new indivisible whole becomes the basic driving unit of professional educational activity of which teaching and/or research are but particular cases. Thus new understanding emerges upon the relationship between research and practice characterized by their mutual reinforcement. These revolutionizing changes within the profession facilitate changes in social status of teachers into teachers-researchers, what in turn increases societal respect and appreciation for the teaching profession, what in the next turn, as demonstrated by the example of Finland, significantly increases understanding and mastery of mathematics by the students.